New exotics in the double beta decay contributions zoo [†]

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Abstract. We discuss the potential of neutrinoless double beta decay for testing Lorentz invariance and the weak equivalence principle as well as contributions from dilaton exchange gravity in the neutrino sector. While neutrino oscillation bounds constrain the region of large mixing of the weak and gravitational eigenstates, we obtain new constraints on violations of Lorentz invariance and the equivalence principle from neutrinoless double beta decay, applying even in the case of no mixing. Double beta decay thus probes a totally unconstrained region in the parameter space.

1. Introduction

Special relativity and the equivalence principle can be considered as the most basic foundations of the theory of gravity. However, string theories allow for or even predict the violation of these laws (see [1, 2] and references therein). Many experiments hunt for these exotics (figure 1), which have been tested to a very high level of accuracy [3] for ordinary matter - generally for quarks and leptons of the first generation. These precision tests of local Lorentz invariance – violation of the equivalence principle

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Figure 1. *The hunt for exotics is still going on and may be successful in neutrinoless double beta decay*

should produce a similar effect [4] – probe for any dependence of the (non–gravitational) laws of physics on a laboratory's position, orientation or velocity relative to some preferred frame of reference, such as the frame in which the cosmic microwave background is isotropic.

A typical feature of the violation of local Lorentz invariance (VLI) is that different species of matter have a characteristical maximum attainable velocity. This can be tested in various sectors of the standard model through vacuum Cerenkov radiation [5], photon decay [6], neutrino oscillations [7, 10, 11, 12, 13, 14] and $K-$physics [8, 9]. In this article we extend these arguments to derive new constraints from neutrinoless double beta decay.

The equivalence principle implies that spacetime is described by unique operational geometry and hence universality of the gravitational coupling for all species of matter. In the recent years there have been attempts to constrain a possible amount of violation of the equivalence principle (VEP) in the neutrino sector from neutrino oscillation experiments [10, 11, 12, 13, 14]. However, these bounds don't apply when the gravitational and the

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weak eigenstates have small mixing. In the following we point out that neutrinoless double beta decay also constrains VEP. VEP implies different neutrino species to feel different gravitational potentials while propagating through the nucleus and hence the contributions of different eigenvalues don't cancel for the same effective momentum. The main result is that neutrinoless double beta decay can constrain the amount of VEP even if the mixing angle is zero, i.e. if only the weak equivalence principle is violated, for which no bound exists at present.

2. Violations of Lorentz invariance

For sake of clarity we formulate the problem for a two generation scenario involving ν_e and ν_x with $x = \mu, \tau, s$. Neutrinos of different species may have different maximum attainable velocities if there is violation of local Lorentz invariance (VLI) and hence violation of special relativity [6]. We first assume that the weak eigenstates cannot be diagonalized simultaneously with the velocity eigenstates and the neutrinos are relativistic point particles. The effective Hamiltonian in the weak basis $[\nu_e \quad \nu_x]$ is

$$H = U_m H_m U_m^{-1} + U_v H_v U_v^{-1}. \tag{1}$$

In absence of VLI the neutrino mass matrix in the mass basis $[\nu_1 \quad \nu_2]$ is given by

$$H_m = \frac{(M_m)^2}{2p} = \frac{1}{2p} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}^2 \tag{2}$$

and the VLI part of the hamiltonian as

$$H_v = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} p, \tag{3}$$

to leading order in $\bar{m}^2/p^2$. Here p denotes the momentum and $\bar{m}$ the average mass, and for any quantity X we define $\delta X \equiv (X_1 - X_2)$, $\bar{X} = (X_1 + X_2)/2$.

In the absence of VLI, i.e. if the special theory of relativity is valid, $v_i = 1$, and H_v simply becomes the momentum of the neutrinos. Here we are interested in a single neutrino beam (for neutrino oscillation experiments) or a single virtual neutrino propagating inside the nucleus with a particular momentum. For this reason we assume the momenta of both the neutrinos are p. Then v_i corresponds to the maximum attainable velocity of the corresponding momentum eigenstates. Hence $v_1 - v_2 = \delta v$ is a measure of VLI in the neutrino sector. As typical or "standard" maximum attainable velocity $\frac{v_1+v_2}{2} = 1$ is assumed. All previous bounds on this quantity δv in the neutrino sector were derived from neutrino oscillation experiments and

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for that reason these bounds are valid only for large gravitational mixing. As we shall point out, neutrinoless double beta decay can constrain δv even if the mixing angle vanishes.

We shall not consider any CP violation, and hence H_m and H_v are real symmetric matrices and U_m and U_v are orthogonal matrices $U^{-1} = U^T$. They can be parametrized as $U_i = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}$, where θ_i represents weak mixing angle θ_m or velocity mixing angle θ_v. We can now write down the weak Hamiltonian H_w in the basis $[\nu_e \ \ \nu_x]$, in which the charged lepton mass matrix and the charged current interaction are diagonal:

$$H = pI + \frac{1}{2p}\begin{pmatrix} M_+ & M_{12} \\ M_{12} & M_- \end{pmatrix}^2.$$

Here I is the identity matrix and

$$
\begin{aligned}
M_+ \ \ = \ \ & \bar{m} \pm \frac{cos2\theta_m}{2}\delta m \\
& \pm \frac{p^2}{\bar{m}}\delta v \left(\frac{\cos 2\theta_v}{2} - \frac{\delta m}{4\bar{m}}\cos 2(\theta_m - \theta_v) \right)
\end{aligned}
\tag{4}
$$

is the double beta observable in the presence of VLI. In the mass mechanism of neutrinoless double beta decay, the half life

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{\langle m \rangle^2}{m_e^2}G_{01}|ME|^2 \tag{5}$$

is proportional to the effective neutrino mass $\langle m \rangle = m_{ee} = M_+$. Here ME denotes the nuclear matrix element $ME = M_F - M_{GT}$, G_{01} corresponds to the phase space factor defined in [15] and m_e is the electron mass. The double beta observable can be written as

$$
\begin{aligned}
<m> \ \ = \ \ & \sum_i U_{ei}^2 m_i = m_1 \cos^2\theta_w + m_2 \sin^2\theta_w \\
= \ \ & \bar{m} + \frac{1}{2}\delta m \cos 2\theta_w.
\end{aligned}
\tag{6}
$$

If $m_{ee} = 0$, the two physical eigenstates with eigenvalues m_1 and m_2 will contribute to the neutrinoless double beta decay by an amount $U_{e1}^2 m_1$ and $U_{e2}^2 m_2$, respectively, which cancels each other. However, if these two physical states have different maximum attainable velocities, corresponding to VLI, this cancellation will not be exact for the same cut–off effective momentum in the neutrino propagator. As a result, even if $m_{ee} = 0$, a non-vanishing contribution to neutrinoless double beta decay can exist, which is proportional to the amount of VLI. In this case the double beta observable is given by M_+ in (4). From (4) it can easily be seen that in

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the region of maximal mixing, $\cos 2\theta_v = 0$, the double beta decay rate vanishes. Thus neutrinoless double beta decay doesn't constrain the amount of VLI for maximal mixing. However, if the mixing approaches zero, the most stringent bound from neutrinoless double beta decay is obtained. In this case $\delta v/2$ can be understood as deviation from the standard maximum attainable velocity $\bar{v}$. Obviously, for the case of vanishing mixing neutrino oscillation experiments cannot give any bound on the amount of VLI: Allowing only for VLI without mixing will not imply neutrino oscillations.

To give a bound on VLI in the small mixing region (including $\theta_v = \theta_m \simeq 0$) we assume conservatively $\langle m \rangle \simeq 0$. We also assume $\delta m \leq \bar{m}$, and thus $\frac{\delta m}{4\bar{m}}$ may be neglected. Due to the p^2 enhancement the nuclear matrix element of the mass mechanism have to be replaced by $\frac{m_p}{R} \cdot (M'_F - M'_{GT})$ with the nuclear radius R and the proton mass m_p, which has been calculated in [16]. Inserting the recent conservative half life limit obtained from the Heidelberg–Moscow experiment [17], $T_{1/2}^{0\nu\beta\beta} > 1.8 \cdot 10^{25} y$ (90%C.L.), a bound on the amount of VLI as a function of the average neutrino mass $\bar{m}$ can be given. The most reliable assumption for $\bar{m}$ is obtained from the cosmological bound $\sum_i m_i < 40$ eV [18], i.e., $\bar{m} < 13$ eV for three generations, implying a bound of [20]

$$\delta v < 4 \times 10^{-16} \quad \text{for} \quad \theta_v = \theta_m \simeq 0.$$

In figure 2 the bound implied by double beta decay is presented for the entire range of $\sin^2 2\theta_v$ and compared with bounds obtained from neutrino oscillation experiments in [12]. It should be stressed also that the GENIUS proposal of the Heidelberg group [19] could improve these bounds by about 1–2 orders of magnitude [20].

3. Violations of the equivalence principle

In the following we present the formalism for violation of the equivalence principle (VEP). While in the final expression the amount of VLI just will be replaced by the corresponding term of VEP, the effects may have a totally different origin. In a linearized theory the gravitational part of the Lagrangian to first order in a weak gravitational field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ ($h_{\mu\nu} = 2\frac{\phi}{c^2} diag(1,1,1,1)$) can be expressed as $\mathcal{L} = -\frac{1}{2}(1 + g_i)h_{\mu\nu}T^{\mu\nu}$, where $T^{\mu\nu}$ is the stress-energy in the gravitational eigenbasis. In presence of VEP the couplings g_i may differ. Assuming only violation of the weak equivalence principle, the gravitational interaction is diagonal. In this case there does not exist any bound on the amount of VEP in the neutrino sector. We point out that this region of the parameter space is most restrictively bounded by neutrinoless double beta decay.
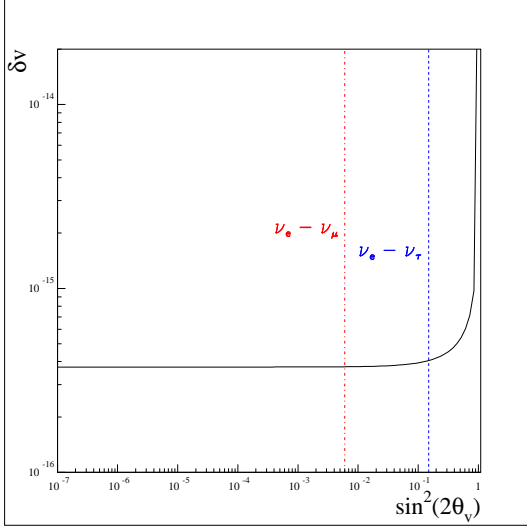
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Figure 2. *Double beta decay bound (solid line) on violation of Lorentz invariance in the neutrino sector, excluding the region to the upper left. Shown is a double logarithmic plot in the δv–$\sin^2(2\theta)$ parameter space. The bound becomes most stringent for the small mixing region, which has not been constrained from any other experiments. For comparison the bounds obtained from neutrino oscillation experiments (from [10]) in the $\nu_e - \nu_\tau$ (dashed lines) and in the $\nu_e - \nu_\mu$ (dashed-dotted lines) channel, excluding the region to the right, are shown.*

The effective Hamiltonian in the weak basis again can be written as

$$H = pI + U_m H_m U_m^{-1} + U_G H_G U_G^{-1}, \qquad (7)$$

with H_m given in (2) and

$$
\begin{aligned}
H_G &= \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \\
&= \begin{pmatrix} -2(1+g_1)\phi(p + \frac{\bar{m}^2}{2p}) & 0 \\ 0 & -2(1+g_2)\phi(p + \frac{\bar{m}^2}{2p}) \end{pmatrix}
\end{aligned} \qquad (8)
$$

to first order in $\bar{m}^2/p^2$. The resulting double beta decay observable is

$$
\begin{aligned}
M_+ &= \bar{m} \pm \frac{\cos 2\theta_m}{2}\delta m \\
&\pm \frac{p}{\bar{m}}\delta G \left(\frac{\cos 2\theta_G}{2} - \frac{\delta m}{4\bar{m}}\cos(\theta_m - \theta_v) \right). \qquad (9)
\end{aligned}
$$

Compared to VLI, the expression remains unchanged for the case of VEP, except for replacing δv by $\frac{1}{p}\delta G$. Again, $\bar{g} = \frac{g_1+g_2}{2}$ can be considered

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as the standard gravitational coupling, for which the equivalence principle applies.

Thus the discussion of VLI can be directly translated to the VEP case and the bound from neutrinoless double beta decay for $\theta_v = \theta_m = 0$ is now given by [20]

$$\phi\delta g < 4 \times 10^{-16} \text{ (for } \bar{m} < 13\text{eV)} \tag{10}$$

In this case, $\delta G = p\phi\delta g$, where ϕ is the background Newtonian gravitational potential on the surface of the earth. A natural choice for ϕ would be the earth's gravitational potential ($\sim 10^{-9}$), but another well motivated choice could be the potential due to all forms of distant matter. Unlike the case of VLI, the bound on the VEP will depend on what one chooses for the Newtonian potential ϕ. For this reason, here we only present the combined bound on $\phi\delta g$.

4. Dilaton exchange gravity

Recently it has been argued by Damour and Polyakov [21] that string theory may lead to a new scalar type gravitational interaction via couplings of the dilaton field and subsequently its consequence to neutrino oscillation has been studied [23]. Damour and Polyakov have shown that the massless dilaton interaction modifies the gravitational potential energy and there is an additional contribution from spin-0 exchange, which results in a scalar type gravitational interaction [21]. The resulting theory is of scalar-tensor type with the two particle static gravitational energy

$$V(r) = -G_N m_A m_B (1 + \alpha_A \alpha_B)/r. \tag{11}$$

Here G_N is Newton's gravitational constant and α_j denotes the couplings of the dilaton field ϕ to the matter field ψ_j, leading to a gravitational energy of

$$L = m_j \alpha_j \overline{\psi_j} \psi_j \phi. \tag{12}$$

Thus the modified effective mass matrix of the neutrinos is now given by [23]

$$m^{(*)} = m - m\alpha\phi_c \tag{13}$$

where the classical value of the dilaton field $\phi_c = \phi_N \alpha_{ext}$ is characterized by the α value of the bulk matter producing it and a static matter distribution proportional to the Newtonian potential ϕ_N is assumed.

The effective mass squared difference

$$\Delta m^{(*)2} = -2\bar{m}^2 \phi_N \alpha_{ext} \delta\alpha \tag{14}$$

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(for almost degenerate masses $m_1 \sim m_2 \sim \bar{m}$) gives rise to neutrino oscillation. The corresponding effect for $0\nu\beta\beta$ decay is obtained by replacing $\delta g^S = 2\delta\alpha^S \bar{m}^2$. Comparing the arguments in the oscillations propabilities we get [24]

$$M_+ = \bar{m}\alpha_{ext}\Phi_N \delta\alpha \frac{\cos(2\theta_G)}{2}. \tag{15}$$

In this case it is difficult to obtain any bound from neutrino experiments, since for α_{ext} only upper bounds exist. To get an idea of the constraints obtainable from neutrino experiments, if in the future α_{ext} is known, according to ref. [23] we assume $\phi_N = 3 \cdot 10^{-5}$, $\alpha_{ext} = \sqrt{10^{-3}}$ and $\bar{m} = 2.5$ eV (as an upper bound obtained from tritium beta decay experiments [25]). In this case the quantity $\delta\alpha$ can not be constrained from neutrinoless double beta decay.

5. A comment on Halprin's critics

In ref [22] it is claimed that neither violations of Lorentz invariance nor violations of the equivalence principle may give sizable contributions to neutrinoless double beta decay. The argument discussed is the following: One considers the neutrino propagator

$$\int d^4 q \frac{e^{-iq(x-y)}\langle m\rangle c_a^2}{m^2 c_a^4 - q_0^2 c_a^2 + \vec{q}^2 c_a^2} \tag{16}$$

with the standard $0\nu\beta\beta$ observable $\langle m\rangle$, the neutrino four momentum q and the characteristic maximal velocity c_a. If one would neglect now q_0 and m in the denominator, c_a drops out and the decay rate is independent of c_a. The discussion of VEP makes use of the same argument.

However, in [20] it has been shown starting from the Hamiltonian level that the propagator (or the $0\nu\beta\beta$ observable) is changed itself if one allows for Lorentz invariance violation. Since

$$
\begin{aligned}
H &= \vec{q}c_a + \frac{m^2 c_a^4}{2\vec{q}c_a} \\
&= \vec{q}I + \frac{m^{(*)2}c_a^4}{2\vec{q}c_a} \tag{17}
\end{aligned}
$$

with $c_a = I + \delta v$ and $m^{(*)2} = m^2 + 2\vec{q}^2 c_a \delta v$ an additional contribution to the effective mass is obtained $\propto \vec{q}^2 \delta v$. This mass-like term has a $\vec{q}^2$ enhancement and is not proportional to the small neutrino mass. This consideration answers also the frequently asked question "What is the source of lepton number violation?" in this mechanism. Comparable to a usual mass term, which can be both of Majoran type as well as Dirac type the mass-like term $2\vec{q}^2 c_a \delta v$ can be of Majorana type and act as the source of lepton number violation in this context.

6. Summary

We discussed the potential of neutrinoless double beta decay searches for exotic phenomena such as violations of Lorentz invariance, the equivalence principle and contributions of dilaton exchange gravity. We pointed out that neutrinoless double beta decay can constrain the amount of VLI or VEP. In particular, when the mixing of the gravitational eigenstates vanishes, the bounds from neutrinoless double beta decay become most stringent, while this region is not constrained by any other experiments.

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